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Some Properties of the Cosine Lomax Distribution with Applications

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Abstract

This paper introduces a new trigonometric distribution, named the Cosine Lomax (CLM) distribution. This distribution is a combination of the Lomax distribution and the Cosine-G family of distributions. Among the many mathematical moments, moment-generating function, incomplete moments, and the quantile function. We try to determine the model parameters using the maximum likelihood estimation method. Simulation studies evaluate the effectiveness of maximum likelihood estimators using bias and root mean square error. We applied the CLM distribution to two real-world datasets and tested for consistency using the Akaike information criterion, the consistent AIC, the Hannan-Quinn information criterion, the Bayesian information criterion, the Kolmogorov-Smirnov p-value, Cramer Von-Mises, and Andersen-Darling. The CLM model fared better when compared to the following distributions: Lomax, Inverse Lomax, Inverse Weibull, Burr Type X, Rayleigh, and Exponential, as well as measures of data set fit.

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1. INTRODUCTION

Parametric statistical distributions often model samples of data sets derived from many sources, including experiments, observational studies, surveys, and more. The primary goal of the majority of these models is to determine which probability distribution adequately depicts the dataset's underlying structure [1]. However, as no model can account for all possible datasets, new probability distributions have to be developed. Adding a family of distributions to classical distributions is a common way to generate new distributions. Many authors have presented various compounded distributions in recent publications by [2-6]. More compound distributions are always needed since no model can suit all datasets. The primary subject of this investigation is the cosine family of distributions, a subset of trigonometric distributions introduced by [7]. A trigonometric family is preferred because of their intrinsic flexibility, which allows them to be more effectively compounded with any baseline distribution. Classical distributions have become more flexible by combining with a trigonometric family, as shown in [8-11].
Following the definition in [7], the CDF of the Cosine-G family of probability distributions is as follows:

$$F(x) = 1 - \cos\left\{\frac{\pi}{2} H(x)\right\} \tag{1}$$

With PDF given by

$$f(x) = \frac{\pi}{2} h(x) \sin\left\{\frac{\pi}{2} H(x)\right\} \tag{2}$$

Where $h(x)$ and $H(x)$ are the PDF and CDF of any baseline distribution.

The objective of this work is to combine the cosine family of distributions with the two parameter Lomax distribution, as introduced [12]. Our goal is to combine these two distributions in order to produce a versatile distribution that can accurately match real-world datasets.

2. THE PROPOSED COSINE LOMAX DISTRIBUTION

The CDF and pdf of the Lomax distribution is defined as:

$$H(x) = 1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha} \tag{3}$$

And

$$h(x) = \frac{\alpha}{\lambda}\left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)} \tag{4}$$

The CDF of the proposed Cosine Lomax (CLM) distribution is derived by substituting (3) into (4), and it is given as follows:

$$F(x) = 1 - \cos\left\{\frac{\pi}{2}\left[1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \tag{5}$$

The corresponding PDF of the CLM distribution is derived by substituting (3) and (4) into (2), which is given as:

$$f(x) = \frac{\pi\alpha}{2\lambda}\left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)} \sin\left\{\frac{\pi}{2}\left[1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \tag{6}$$

Where x is a random variable that is greater than one, $\lambda > 0$ is the scale parameter, and $\alpha > 0$ is the shape parameter.

Equations (7) and (8) provide the presentation of the survival function S(x) and hazard function r(x).

$$S(x) = \cos\left\{\frac{\pi}{2}\left[1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \tag{7}$$

$$r(x) = \frac{\pi\alpha}{2\lambda}\ \left(1 + \frac{x}{\lambda}\right)^{-(\alpha+1)} \tan\left\{\frac{\pi}{2}\left[1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \tag{8}$$

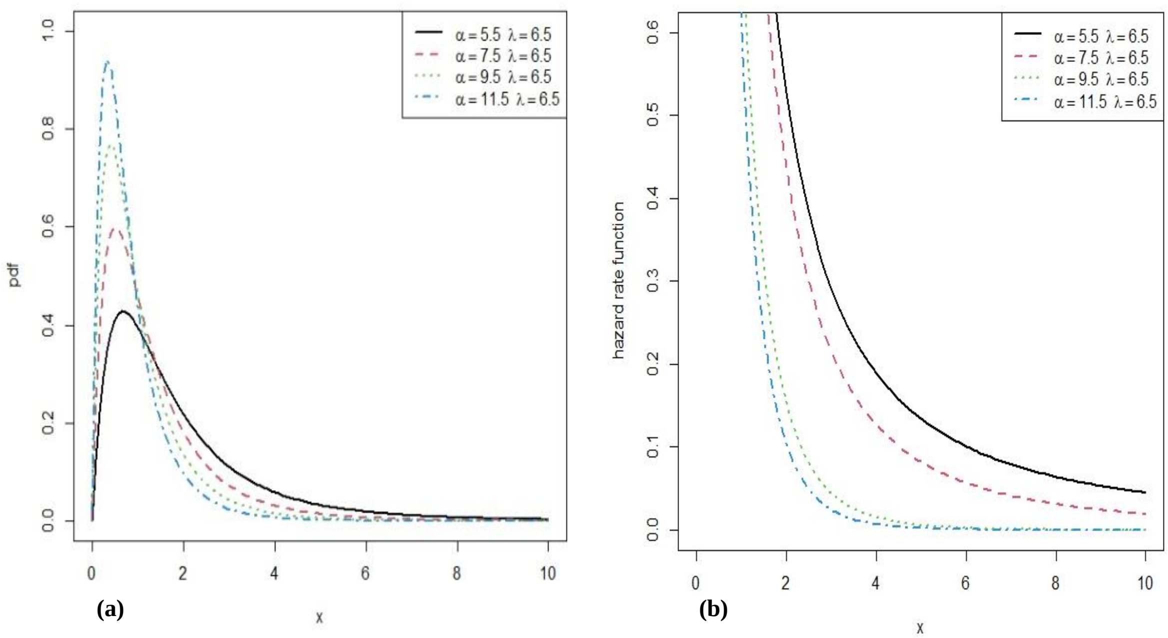

Figure 1. (a) PDF of the CLM, (b) r(x) plot of CLM distribution

3. MATHEMATICAL PROPERTIES

3.1 The linear representation of the CLM distribution

In this part, we provide a linear representation for the CLM distribution, allowing us to effectively compute the mathematical properties of the proposed model using Maclaurin series expansion:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \tag{9}$$

And,

$$\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \tag{10}$$

The CDF and PDF of the CLM distribution can be represented as a mixture.

$$\cos\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\} = \sum_{p=0}^{\infty} \frac{(-1)^p \pi^{2p}}{(2p)!\,2^{2p}}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2p} \tag{11}$$

Then

$$F(x) = 1 - \sum_{p=0}^{\infty} \frac{(-1)^p \pi^{2p}}{(2p)!\,2^{2p}}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2p} \tag{12}$$

Using binomial expansion [13]

$$\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]^{2p} = \sum_{q=0}^{\infty} (-1)^q \binom{2p}{q}\left(1+\frac{x}{\lambda}\right)^{-\alpha q} \tag{13}$$

$$F(x) = 1 - \sum_{p=q=0}^{\infty} \frac{(-1)^{p+q} \pi^{2p}}{(2p)!\,2^{2p}}\binom{2p}{q}\left(1+\frac{x}{\lambda}\right)^{-\alpha q} \tag{14}$$

We have obtained by taking the derivative of the previous equation (14) with respect to x.

$$f(x) = \sum_{p=q=0}^{\infty} \frac{(-1)^{p+q} \pi^{2p}}{(2p)!\,2^{2p}}\binom{2p}{q}\frac{\alpha q}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-\alpha q-1}$$

$$= \sum_{q=0}^{\infty} ¥_{p,q}\, h_q(x) \tag{15}$$

Where $p = 0,1,2,\dots,\infty, q = 0,1,2,\dots,\infty$, $¥_{p,q} = \sum_{p=0}^{\infty} \frac{(-1)^{p+q}\pi^{2p}}{(2p)!2^{2p}}\binom{2p}{q}$, and $h_q(x)$ are called the PDF of the Lomax distribution with shape parameter αq and scale parameter λ.

3.2 Moment

Moments serve multiple functions in statistical analyses, encompassing tasks such as parameter estimation, evaluating skewness and kurtosis, and generating descriptive statistics. They offer a compact overview of the key characteristics of the underlying data distribution [14]. A specific formula enables the calculation of these moments:

$$\acute{\mu}_r = E(X^r) = \int_0^{\infty} x^r f(x)dx \tag{16}$$

Substitution equation (15) into equation (16)

$$\acute{\mu}_r = E(X^r) = \sum_{q=0}^{\infty} ¥_{p,q} \int_0^{\infty} x^r h_q(x)dx \tag{17}$$

Then

$$\acute{\mu}_r = E(X^r) = \sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^r \Gamma(r+1)\Gamma(\alpha q - r)}{\Gamma(\alpha q + 1)} \tag{18}$$

Equation (13) yields the CLM distribution $\acute{\mu}_1, \acute{\mu}_2, \acute{\mu}_3, and\ \acute{\mu}_4$ as follows.

$$\acute{\mu}_1 = \sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^1 \Gamma(2)\Gamma(\alpha q - 1)}{\Gamma(\alpha q + 1)} \tag{19}$$

$$\acute{\mu}_2 = \sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^2 \Gamma(3)\Gamma(\alpha q - 2)}{\Gamma(\alpha q + 1)} \tag{20}$$

$$\acute{\mu}_3 = \sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^3 \Gamma(4)\Gamma(\alpha q - 3)}{\Gamma(\alpha q + 1)} \tag{21}$$

$$\acute{\mu}_4 = \sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^4 \Gamma(5)\Gamma(\alpha q - 4)}{\Gamma(\alpha q + 1)} \tag{22}$$

And by using equations (19), (20), (21), and (22) we can find the skewness and kurtosis.

3.3 Moment Generating Function

The moment generating function for a random variable X is defined as the expected value of the exponential function e^{tx}, where t represents a real-valued parameter. The moment generating function of the proposed CLM distribution is obtained as follows [15]:

$$M_x(t) = E(e^{tx}) = \int_0^{\infty} e^{tx} f(x) dx \tag{23}$$

$$M_x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!} (\acute{\mu}_r) = \sum_{k=0}^{\infty} \frac{t^k}{k!} \left(\sum_{q=0}^{\infty} ¥_{p,q} \frac{\alpha q \lambda^r \Gamma(r+1)\Gamma(\alpha q - r)}{\Gamma(\alpha q + 1)} \right) \tag{24}$$

3.4 Incomplete Moments

The distribution of the CLM has an incomplete moment that is provided by [16]:

$$\acute{\mu}_r(u) = \int_0^{u} x^r f(x) dx \tag{25}$$

Substitution equation (15) into equation (25)

$$\acute{\mu}_r(z) = \sum_{q=0}^{\infty} ¥_{p,q} \int_0^{z} x^r h_q(x) dx \tag{26}$$

Then

$$\acute{\mu}_r(u) = \sum_{q=0}^{\infty} ¥_{p,q}\, \alpha q (-1)^{r-1} \lambda^r B_{-\frac{z}{\lambda}}(r+1, -\alpha q) \tag{27}$$

3.5 Quantile function

The expression for the quantile function Q(u) of the random variable X can be obtained by inverting the CDF of the CLM model [17]:

$$Q(u) = \lambda \left[\left(1 - \frac{\cos^{-1}(1-u)}{\pi/2} \right)^{-\frac{1}{\alpha}} - 1 \right] \tag{28}$$

on the unit interval [0,1], we find the uniform random variable u.

4. MAXIMUM LIKELIHOOD ESTIMATION

We estimated the parameters of the newly proposed model using the maximum likelihood estimate method, as follows: We can express the likelihood function as [18] by taking the log-likelihood of the proposed model's pdf in equation (6).

$$\ell = n \log\left(\frac{\pi}{2}\right) + n \log \alpha - n \log \lambda - (\alpha + 1) \sum_{i=1}^{n} \log\left(1 + \frac{x}{\lambda}\right) + \sum_{i=1}^{n} \tan\left\{\frac{\pi}{2}\left[1 - \left(1 + \frac{x}{\lambda}\right)^{-\alpha}\right]\right\} \tag{29}$$

Differentiating equation (14) with respect to α gives the following expression:

$$\frac{\partial \ell}{\partial \alpha}=\frac{n}{\alpha}-\sum_{i=1}^{n}\log\left(1+\frac{x}{\lambda}\right)+\sum_{i=1}^{n}\frac{\frac{\pi}{2}\left(1+\frac{x}{\lambda}\right)^{-\alpha}\log\left(1+\frac{x}{\lambda}\right)\sec^{2}\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}}{\tan\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}} \tag{30}$$

Differentiating equation (14) with respect to λ gives the following expression:

$$\frac{\partial \ell}{\partial \lambda}=-\frac{n}{\lambda}+(\alpha+1)\sum_{i=1}^{n}\frac{x}{\lambda^{2}}\left(1+\frac{x}{\lambda}\right)^{-1}-\sum_{i=1}^{n}\frac{\frac{\pi}{2}\frac{\alpha x}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}\sec^{2}\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}}{\tan\left\{\frac{\pi}{2}\left[1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}\right]\right\}} \tag{31}$$

Equations (30) and (31) give the maximum likelihood estimates of the parameters α and λ respectively.

5. SIMULATION STUDY

A simulation study was undertaken to assess the efficacy of the proposed CLM distribution. The goal was to assess the precision of parameter estimations obtained using MLE by evaluating their average, bias, and root mean square error (RMSE) [19]. The quantile function provided in equation (20) was used to produce simulated datasets. Sample sizes of n = 20, 50, 100, 200, 250, 500, and 1000, were considered, with each sample size replicated 1000 times. The parameter combinations tested were λ=0.5, α=0.5; λ=1.5, α=0.4; λ=0.3, α=0.3; and λ=1.2, α=0.6 for each sample size. Table 1 summarizes the estimation results, bias, and root mean square error.

Table 1**.** Simulation Results for CLM distribution parameters using Monte Carlo.

N	Properties	λ=0.5	α =0.5	λ=1.5	α =0.4	λ=0.3	α =0.3	λ=1.2	α =0.6
20	Est	0.8115	0.5954	2.4265	0.4611	0.5223	0.3392	2.0133	0.7438
	Bias	0.3115	0.0954	0.9265	0.0611	0.2223	0.0392	0.8133	0.1438
	RMSE	0.9230	0.2603	2.6392	0.1666	0.6212	0.1066	2.9460	0.4401
50	Est	0.5943	0.5323	1.7969	0.4225	0.3703	0.3149	1.4217	0.6438
	Bias	0.0943	0.0323	0.2969	0.0225	0.0703	0.0149	0.2217	0.0438
	RMSE	0.3074	0.1101	0.9630	0.0794	0.2203	0.0542	0.7229	0.1450
100	Est	0.5445	0.5143	1.6409	0.4099	0.3336	0.3067	1.3072	0.6198
	Bias	0.0445	0.0143	0.1409	0.0099	0.0336	0.0067	0.1072	0.0198
	RMSE	0.1790	0.0675	0.5589	0.0490	0.1250	0.0339	0.4234	0.0881
200	Est	0.5204	0.5068	1.5647	0.4048	0.3151	0.3032	1.2492	0.6094
	Bias	0.0204	0.0068	0.0647	0.0048	0.0151	0.0032	0.0492	0.0094
	RMSE	0.1168	0.0464	0.3663	0.0344	0.0802	0.0238	0.2751	0.0600
250	Est	0.5168	0.5048	1.5533	0.4033	0.3123	0.3021	1.2396	0.6065
	Bias	0.0168	0.0048	0.0533	0.0033	0.0123	0.0021	0.0396	0.0065
	RMSE	0.1051	0.0409	0.3281	0.0303	0.0729	0.0210	0.2465	0.0528
500	Est	0.5054	0.5016	1.5174	0.4011	0.3043	0.3007	1.2126	0.6022
	Bias	0.0054	0.0016	0.0174	0.0011	0.0043	0.0007	0.0126	0.0022
	RMSE	0.0674	0.0279	0.2127	0.0209	0.0467	0.0145	0.1575	0.0357
1000	Est	0.5019	0.5010	1.5060	0.4008	0.3016	0.3006	1.2046	0.6014
	Bias	0.0019	0.0010	0.0060	0.0008	0.0016	0.0006	0.0046	0.0014
	RMSE	0.0478	0.0190	0.1495	0.0141	0.0326	0.0098	0.1121	0.0244

The simulation results shown in Table 1 indicate that the CLM distribution has consistent characteristics. This is shown by the estimations, which approach the genuine parameter values as the sample sizes rise. Furthermore, as sample sizes grow, the declining bias and RMSE provide further evidence of this consistency

6. APPLICATION AND DISCUSSION

Two real-world examples are provided here to demonstrate the versatility of the CLM model in fitting data. The package in the R program uses the MLE approach to estimate the model parameters. Many statistical criteria are used in our study, including the Akaike Information Criterion (AIC), the consistent AIC, the Hannan- Quinn Information Criterion (HQIC), the Kolmogorov-Smirnov (KS) statistic, the P-Value, the Bayesian Information Criterion (BIC), Cramer Von-Mises (W), and Andersen-Darling (A) goodness-of-fit statistics. The ideal model is the one that exhibits higher p-values and lower goodness-of-fit statistics.
Here are several models that we compare to the CLM model: The models we compare include the Lomax distribution [12], the Inverse Lomax distribution [20], the Inverse Weibull distribution [21], the Burr Type X distribution [22], the Rayleigh distribution [23], and the Exponential distribution [24].

Dataset 1: The first data set represents the distances from the transect line for the 68 stakes detected in walking L = 1000 m and searching w = 20 m on each side of the line obtained from [25]. The data set are as follows:

2.0, 0.5, 10.4, 3.6, 0.9, 1.0, 3.4, 2.9, 8.2, 6.5, 5.7, 3.0, 4.0, 0.1, 11.8, 14.2, 2.4, 1.6, 13.3, 6.5, 8.3, 4.9, 1.5, 18.6, 0.4, 0.4, 0.2, 11.6, 3.2, 7.1, 10.7, 3.9, 6.1, 6.4, 3.8, 15.2, 3.5, 3.1, 7.9, 18.2, 10.1, 4.4, 1.3, 13.7, 6.3, 3.6, 9.0, 7.7, 4.9, 9.1, 3.3, 8.5, 6.1, 0.4, 9.3, 0.5, 1.2, 1.7, 4.5, 3.1, 3.1, 6.6, 4.4, 5.0, 3.2, 7.7, 18.2, 4.1.

Table 2. Statistical explanation for the Data 1

Var.	N	Mean	SD	Median	Min	Max	SK	KU
X	68	5.85	4.61	4.45	0.1	18.6	1.02	0.56

Table 2 and Figure 2 the data I and CLM distribution graphs are favorably skewed. The Data I is platykurtic since their KU are smaller than 3.

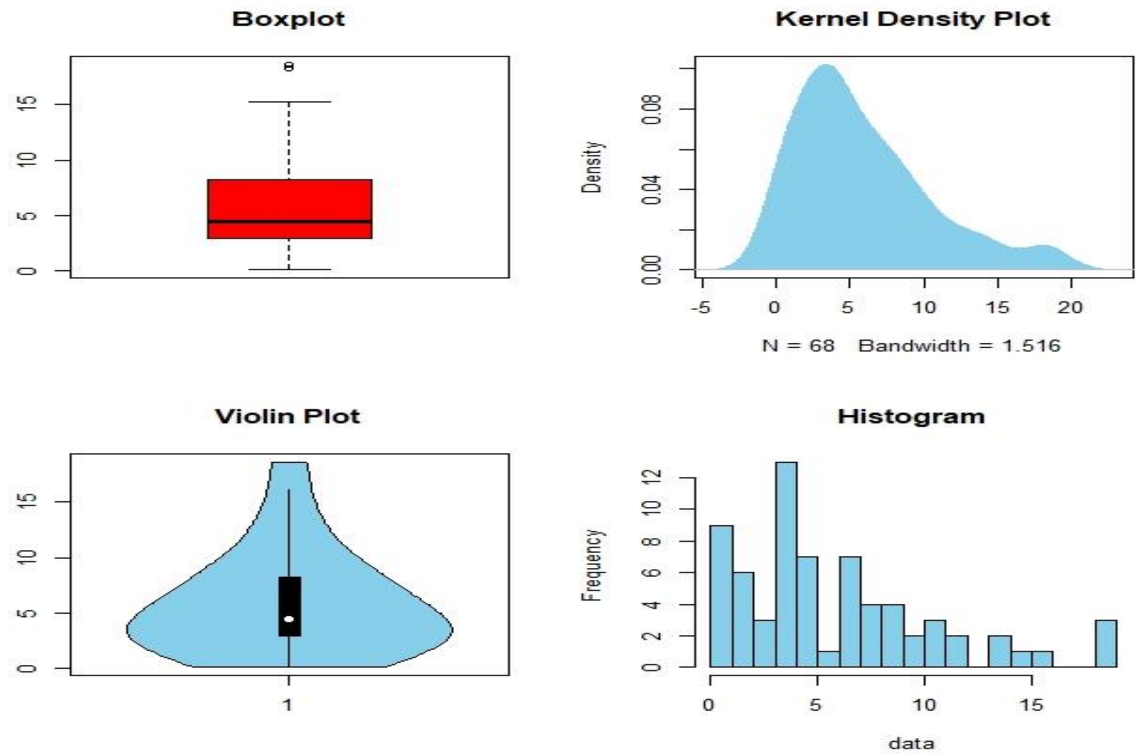

Figure 2. Boxplot, Kernel plot, Violin plot and Histogram of the data 1.

Table 3. MLEs and goodness-of-fit statistic

Statistic	CLM	LM	ILM	IW	BX	R	E
MILS $\hat{\alpha}$	**1.4163**	5.9223	1.8909	1.7265	0.1029	0.0181	0.1708
$\hat{\lambda}$	**4.2607**	3.3926	1.8691	0.7428	0.4725		
-LL	**190.28**	190.58	200.44	209.97	195.74	202.46	198.15
AIC	**384.57**	385.19	404.89	423.94	385.49	406.93	388.30
CAIC	**384.75**	385.38	405.08	424.12	385.67	406.99	388.63
BIC	**389.01**	389.63	409.33	428.38	389.92	409.15	390.52
HQIC	**386.33**	386.95	406.65	425.70	387.24	407.81	389.18
W	**0.1147**	0.9482	0.3804	0.7535	0.4255	0.4563	0.6457
A	**0.5431**	0.6187	2.2844	4.2157	0.7093	0.7093	0.6214
KS	**0.0851**	0.1818	0.1656	0.2217	0.0912	0.2081	0.1554
P-value	**0.7080**	0.0223	0.0479	0.0024	0.6911	0.0055	0.0748

Table 3 contains the largest p-value and the lowest values for the information criteria AIC, CAIC, BIC, and HQIC, as well as the goodness-of-fit statistics W, A, and KS. When testing with data sets consisting of data 1, the CLM model outperforms the alternatives. Figure 3 and the empirical CDF plot show that the CLM model fared better regarding data 1 fit than the LM, ILM, IW, BX, R, and E models.

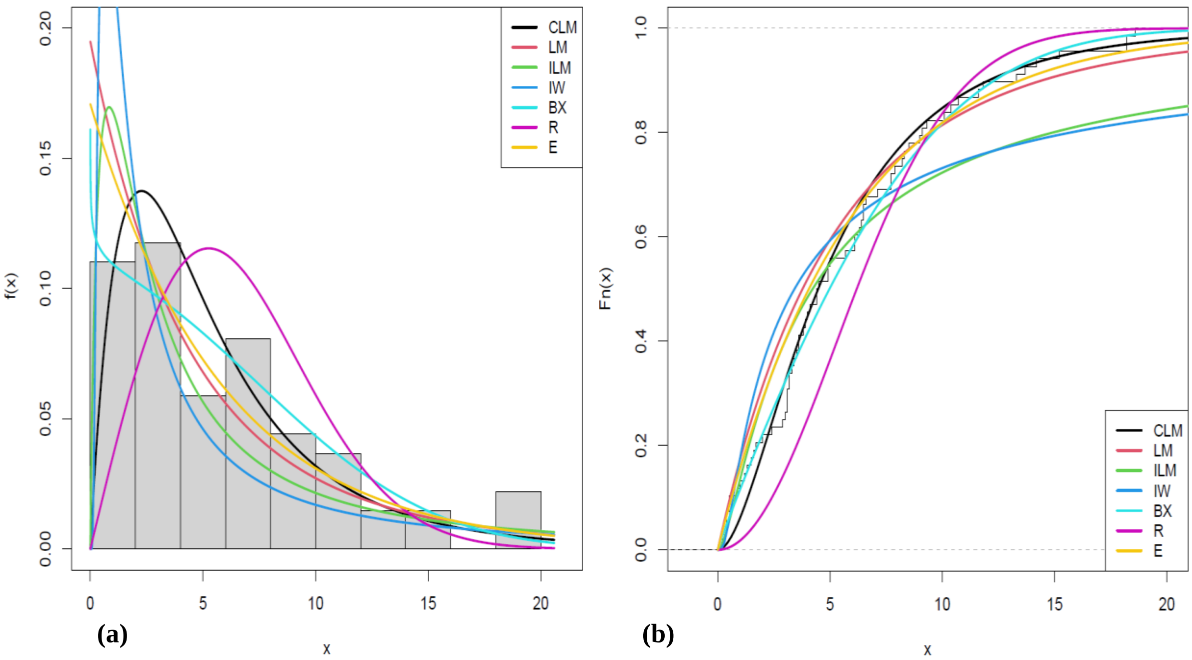

Figure 3. (a) Fitted PDFs for the CLM distribution, (b) Empirical CDF plot for the CLM distribution.

From Figures 3(a) and 3(b), the histogram shows that the CLM distribution has a large peak, and this fits the empirical data well. Table 3 provides the results.

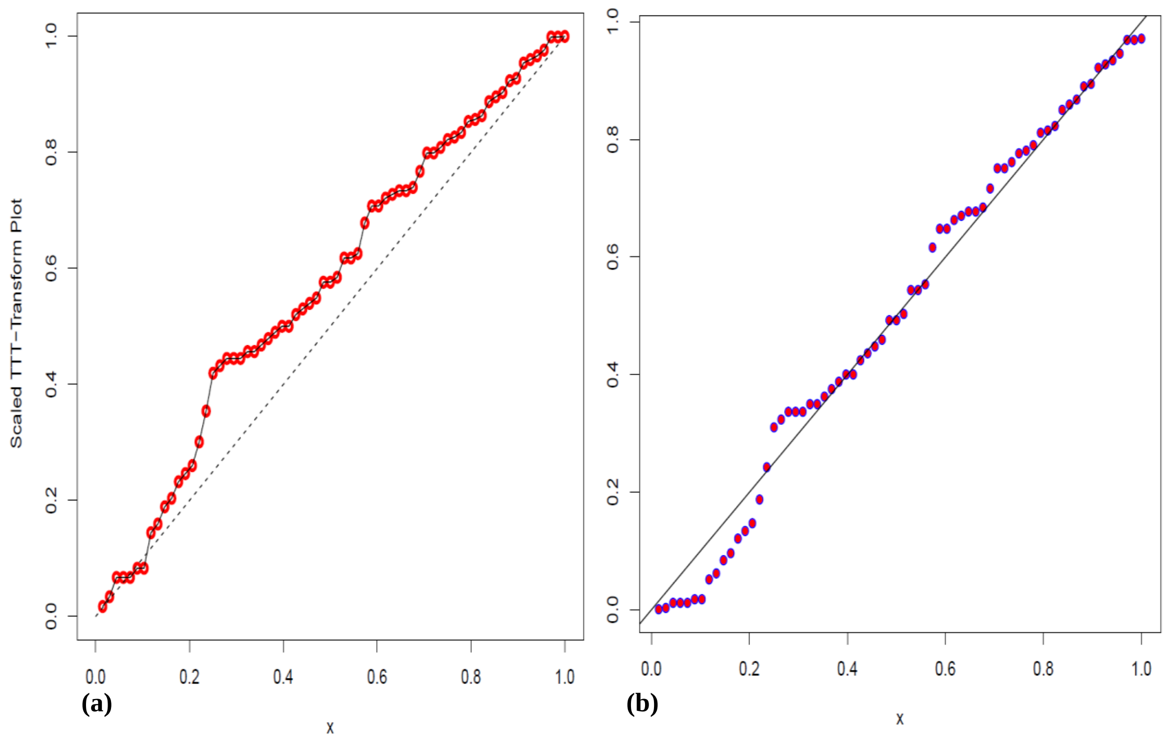

Figure 4. (a) TTT plot for the data 1, (b) PP plot for the CLM distribution.

Figures 4(a) and 4(b) above demonstrate how the new distribution is adaptable when it comes to modeling the empirical data.

Dataset 2: Kevlar 373 and epoxy were subjected to a fatigue fracture life experiment. A stress level of 90% was applied to the material continuously until it failed. The information is presented below [26]:

0.0251,0.0886,0.0891,0.2501,0.3113,0.3451,0.4763,0.5650,0.5671,0.6566,0.6748,0.6751,0.6753,0.7696,0.8375,0.8391,0.8425,0.8645,0.8851,0.9113,0.9120,0.9836,1.0483,1.0596,1.0773,1.1733,1.2570,1.2766,1.2985,1.3211,1.3503,1.3551,1.4595,1.4880,1.5728,1.5733,1.7083,1.7263,1.7460,1.7630,1.7746,1.8275,1.8375,1.8503,1.8808,1.8878,1.8881,1.9316,1.9558,2.0048,2.0408,2.0903,2.1093,2.1330,2.2100,2.2460,2.2878,2.3203,2.3470,2.3513,2.4951,2.5260,2.9911,3.0256,3.2678,3.4045,3.4846,3.7433,3.7455,3.9143,4.8073, 5.4005,5.4435,5.5295,6.5541,9.0960.

Table 4. Statistical Explanation for the Data 2

Var.	N	Mean	SD	Median	Min	Max	SK	KU
X	76	1.96	1.57	1.74	0.03	9.1	1.94	4.95

Table 4 and Figure 5 the data I and CLM distribution graphs are favorably skewed. The Data IN datasets are leptokurtic since their KU are larger than 3.

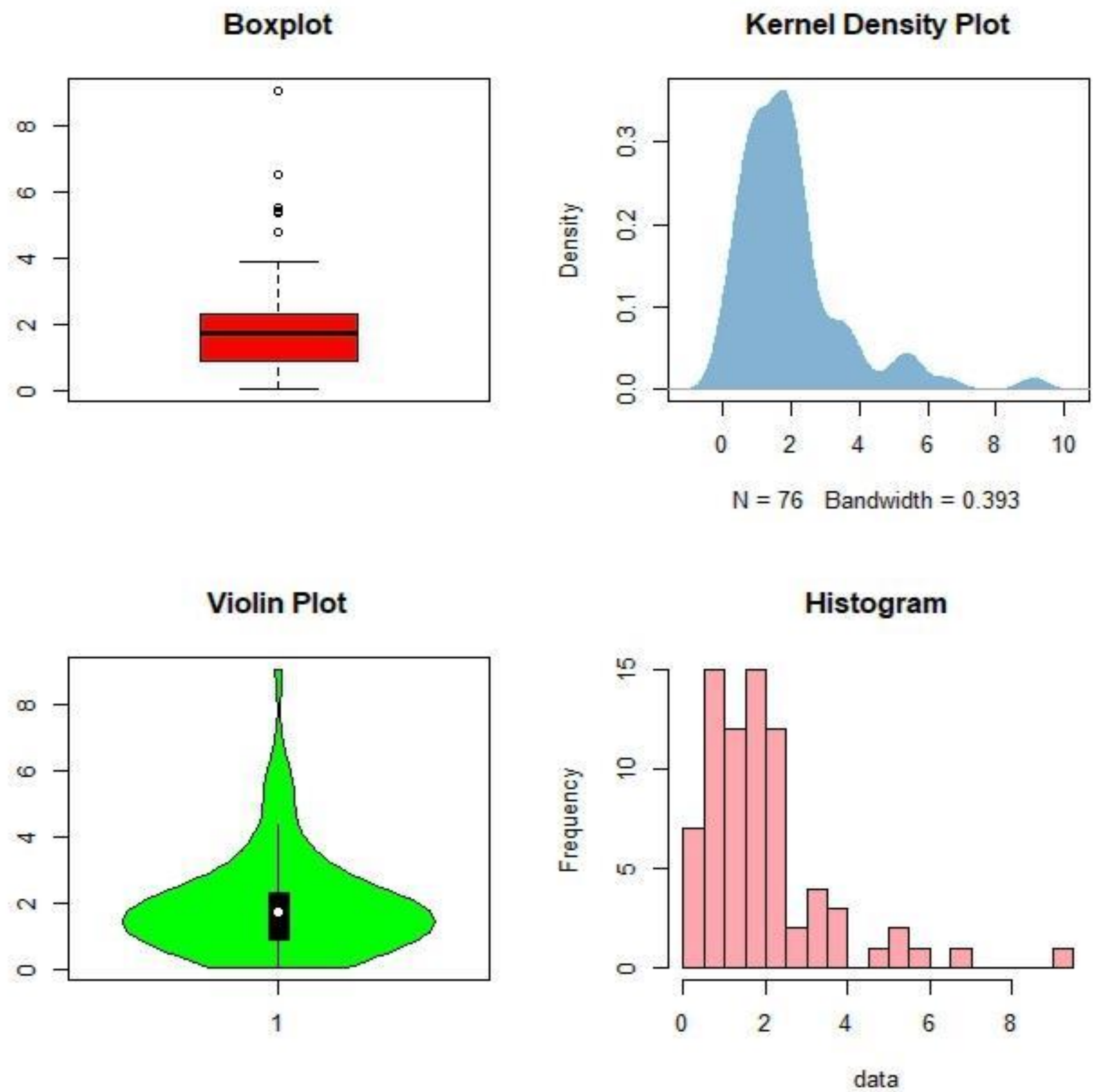

Figure 5. Boxplot, Kernel plot, Violin plot, and Histogram of the data 2.

Table 5. MLEs and goodness-of-fit statistic

Statistic	CLM	LM	ILM	IW	BX	R	E
MILS $\hat{\alpha}$	**3.3946**	1.7949	2.5432	0.8608	0.3165	0.1591	0.5104
$\hat{\lambda}$	**2.1126**	2.6899	0.4966	0.7588	0.5324		
-LL	**122.61**	128.41	139.48	153.53	125.50	137.31	127.11
AIC	**249.22**	260.84	282.96	311.07	255.00	276.63	256.22
CAIC	**249.38**	261.00	283.12	311.24	255.16	276.69	256.28
BIC	**253.88**	265.50	287.62	315.73	259.66	278.97	258.55
HQIC	**251.08**	262.70	284.82	312.94	256.86	277.57	257.16
W	**0.1207**	0.1159	0.4005	0.9167	0.2220	0.2086	0.1192
A	**0.7205**	0.6916	2.4408	5.3394	1.2747	1.2016	0.7073
KS	**0.0861**	0.1781	0.1830	0.1893	0.1551	0.2043	0.1663
P-value	**0.5941**	0.0139	0.0106	0.0073	0.0459	0.0029	0.0263

Table 5 provides the largest p-value and the lowest values for the information criteria AIC, CAIC, BIC, and HQIC and goodness-of-fit statistics W, A, and KS. When testing with data sets consisting of data 2, the CLM model outperforms the alternatives. Figure 6 and the empirical CDF plot show that the CLM model fared better regarding data 2 fit than the LM, ILM, IW, BX, R, and E models.

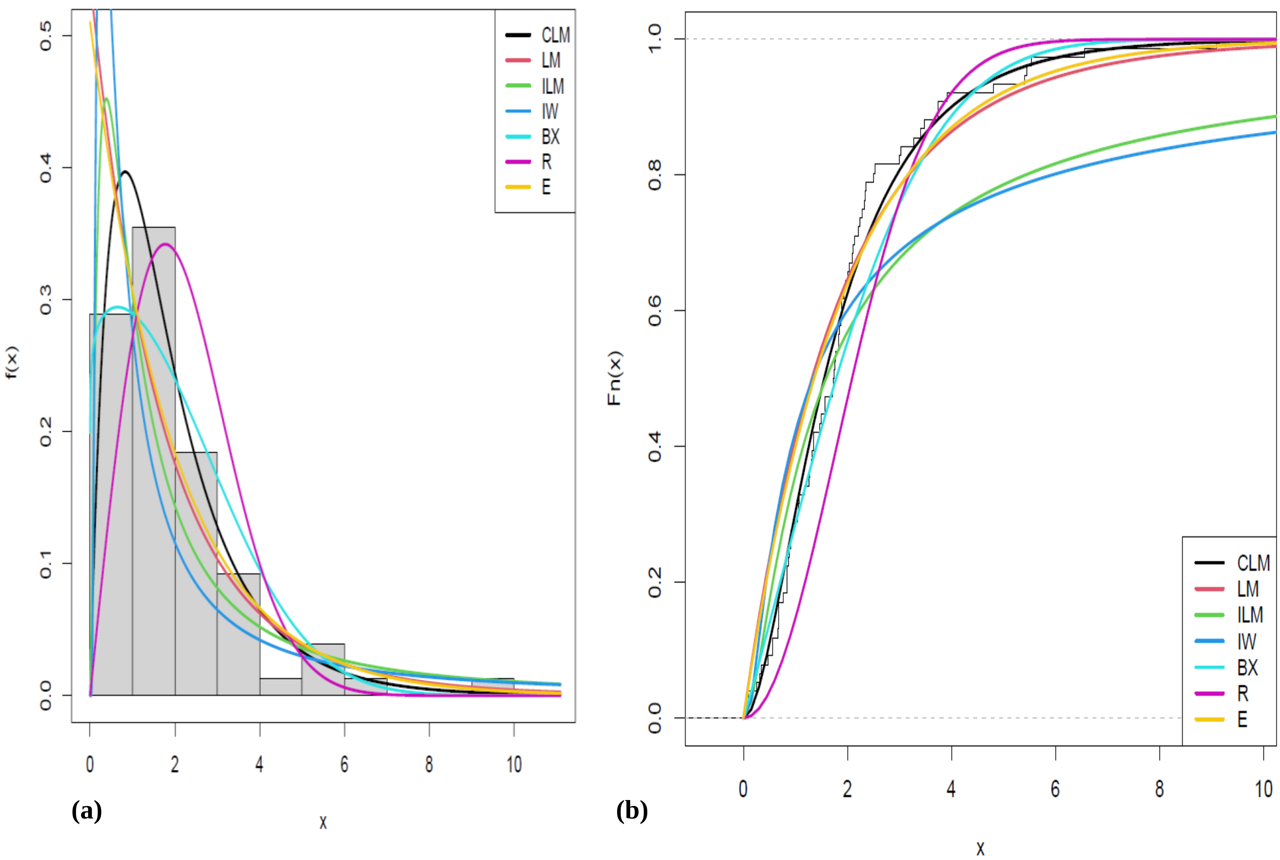

Figure 6. (a) Fitted PDFs for the CLM distribution, (b) Empirical CDF plot for the CLM distribution.

From Figures 6(a), and 6(b) the histogram shows that the CLM distribution has a large peak, and this fits the empirical data well, the results are provided in Table 5.

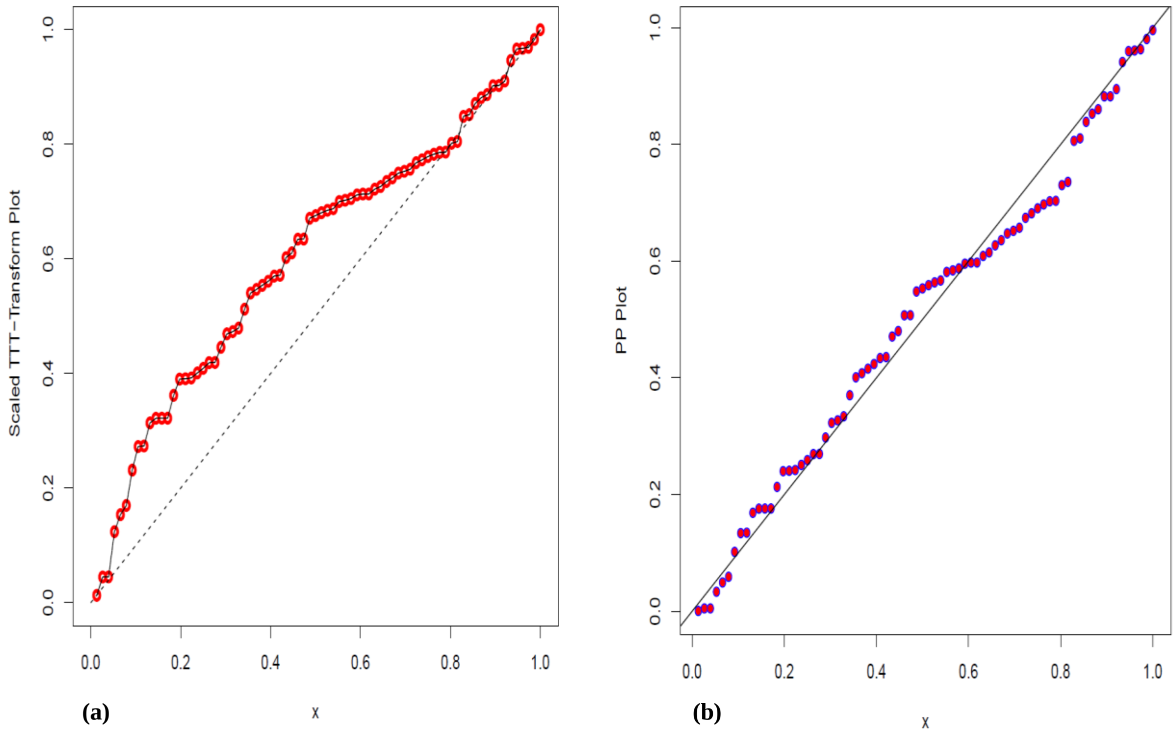

Figure 7. (a) TTT plot for the Data 2, (b) PP plot for the CLM distribution

Figures 7(a) and 7(b) above demonstrate how the new distribution is adaptable when it comes to modeling the empirical data.

7. CONCLUSION AND RECOMMENDATIONS

This paper proposes a new trigonometric distribution and derives some of its mathematical and statistical properties. A simulation study was conducted to investigate the consistency of the model using different parameter values and sample sizes. The simulation's outcome revealed consistent behavior, as evidenced by the reduction in bias and RMSE with larger sample sizes. The possibilities and adaptability of the new distribution are demonstrated using two real-world data sets, which are compared to existing distributions such as the LM, ILM, IW, BX, R, and E models. W, A, KS, AIC, CAIC, HQIC, and P-value are some of the goodness-of-fit statistics used as a comparison basis. According to goodness-of-fit statistics, the CLM distribution outperforms the alternatives in terms of model fit. In order to find a new distribution and estimate distribution parameters effectively, researchers might use this new family.

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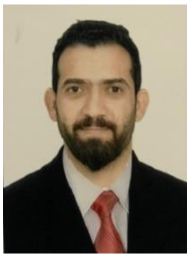

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